AN ANALYTICAL AND EXPERIMENTAL STUDY OF CHAOTIC OSCILLATION IN A PNEUMATIC CYLINDER

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Seal friction of pneumatic cylinders possesses uncertain properties due to the interference of various factors, and it causes random oscillation or stick-slip when the piston is driven at extremely low speed. A deeper understanding of the friction force is required for avoiding this oscillation phenomenon, which results in a deterioration of performance. This paper focuses on analyzing the oscillation phenomenon appearing in time responses of the driving velocity of the piston. In order to investigate aspects of velocity responses in a meter-in and meter-out circuit, we first use an analytical approach with a mathematical model including the friction force under some assumptions. An experimental study is then performed, showing the relationship between an effective sectional area of control valves and oscillatory behavior. It also shows by using some chaos indices, such as Lyapunov exponents, that the behavior of piston velocity turns chaotic under certain conditions in actual situations. In addition, the effects of changes in both supply pressure and load are examined. Based on both theoretical and experimental results, we confirm that the oscillation is caused by the nonlinear friction force.

Keywords: Pneumatic Cylinder, Friction Force, Stick-Slip, Chaotic Oscillation

1 INTRODUCTION

In low-speed driving of pneumatic cylinders, the trade-off between driving force generated by air pressure and seal friction has to be considered to avoid random oscillation or stick-slip. However, frictional properties are unclear because they depend on a number of factors, such as lubrication condition, operating conditions, and interface temperature. In order to acquire knowledge of the friction force, several attempts have been made. Belforte, G.; Raparelli, T.; Velardocchia, M. (1993) performed experimental studies to assess the influence of seal material and dimension, speed, and pressure, etc., on friction force. Schroeder, L.E.; Singh, R. (1993) presented several models of friction force and evaluated them with measured results. But these studies did not bring about a sufficient elucidation regarding a low-speed range.

This paper closely investigates random oscillation appearing in velocity responses when pneumatic cylinders are operated at low speed, and discusses this oscillation's relationship to friction force at the seal. To begin with, numerical simulation of velocity responses is conducted using a model with friction force for a meter-in and meter-out circuit. Next, random

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oscillation is described in relation to an effective sectional area of control valves from systematic experimental results. This oscillation phenomenon can be qualitatively assigned to some regions with the effective sectional area as a parameter, and chaotic oscillation occurs in a region only in experiments. With regard to the chaotic oscillation region, the effects of changes in supply pressure and in load are examined. Finally, it is verified that the dynamic behavior is principally due to transformation of oscillation caused by the nonlinearity of the friction force.

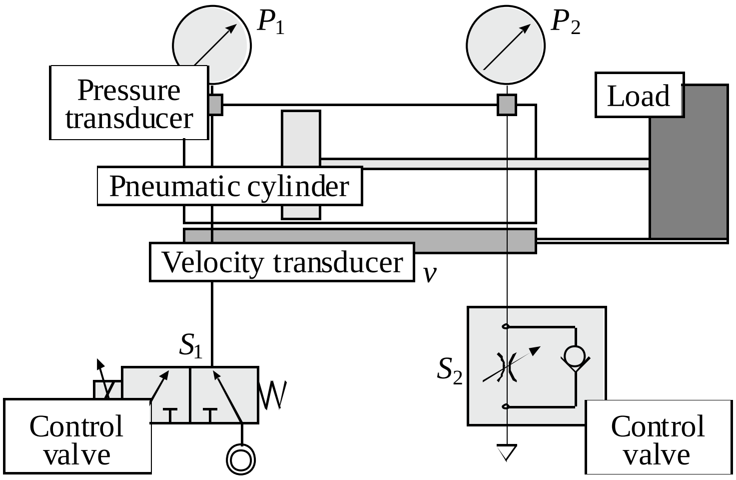


Figure 1: Experimental apparatus

2 EXPERIMENTAL APPARATUS ARRANGEMENT

The structure of the experimental apparatus for driving loads by means of a pneumatic cylinder is illustrated in Fig. 1. A flow rate of supply air to the supply-side cylinder chamber is controlled by a solenoid valve. Air in the rod-side chamber is exhausted through a speed-control valve. The air pressure in both chambers is measured by semiconductor transducers. The driving velocity of the piston is directly obtained from a linear velocity transducer that uses the induced voltage of coils. Their output signals are sent to a PC through an A/D converter, and all the measured values are recorded on it.

3 THEORETICAL ANALYSIS

3.1 Fundamental Equations

A model of the pneumatic drive system shown in Fig. 1 consists of an equation of motion concerning a moving part and equations for pressure changes in cylinder chambers. It is described as follows:

$$m\frac{dv}{dt} = A_1(P_1 - P_a) - A_2(P_2 - P_a) - F_n, \tag{1}$$

$$\frac{dP_1}{dt} = \frac{k\,RT_1}{V_1}\cdot W_1 - \frac{k\,P_1A_1}{V_1}\cdot v, \tag{2}$$

$$\frac{dP_2}{dt} = -\frac{k\,RT_2}{V_2}\cdot W_2 + \frac{k\,P_2A_2}{V_2}\cdot v, \tag{3}$$

where $V_1 = V_{10} + A_1x$ and $V_2 = V_{20} - A_2x.$ Taking a widely used assumption, the friction force is defined as the following equation:

$$F_n = \begin{cases} F_s(v = 0) \\ F_c\,\mathrm{sgn}\,v + Bv(v \neq 0) \end{cases} \tag{4}$$

According to the procedure given by Kawakami, Y.; Noguchi, H.; Kawai, S. (1990), velocity responses are analyzed with the model that includes the friction force of (4). The influence of the friction force on the velocity responses is studied.

3.2 Meter-Out Circuits

In the case of the meter-out circuit, we assume that the pressure in the supply-side cylinder chamber is constant (= supply pressure) and that a rod-side restrictor reaches critical condition. If we use a state equation, a continuity equation, and an energy equation, from (1) ~ (3) we obtain

$$m\frac{d^2v}{dt^2} + \frac{k\,mRT_2C_{d2}}{V_2}\cdot\frac{dv}{dt} + \frac{kA_2^2P_2}{V_2}\cdot v + \frac{dF_n}{dt} + \frac{k\,RT_2C_{d2}}{V_2}\cdot F_n = \frac{k\,A_2R\,T_2C_{d2}}{V_2}\left\{\frac{A_1}{A_2}(P_1 - P_a) + P_a\right\}. \tag{5}$$

Considering infinitesimally small changes from an equilibrium state, equation (5) is linearized as

$$m\frac{d^2(\Delta v)}{dt^2} + (ma + B)\cdot\frac{d(\Delta v)}{dt} + \left(\frac{k\,A_2^2P_{20}}{V_{20}} - a\,B\right)\cdot\Delta v = 0 \tag{6}$$

where $a = (k\,RT_{20}C_{d2} - k\,A_2v_0)/V_{20}$. Besides, by replacing with a_1(= $ma + B$) and a_2(= $k\,A_2^2P_{20}/V_{20} - a\,B$), we get

$$\frac{d^2(\Delta v)}{dt^2} + \frac{a_1}{m}\cdot\frac{d(\Delta v)}{dt} + \frac{a_2}{m}\cdot\Delta v = 0. \tag{7}$$

The system (7) is second-order and its damping coefficient is expressed as

$$V = a_1 \Big/ \left(2\sqrt{m a_2}\right). \tag{8}$$

Since the sign of the denominator of (8) is positive, the sign of a_1 determines system damping. From (5), we have (assuming $A_1 \approx A_2$)

$$a_1 = \frac{k m R T_{20} C_{d2}}{V_{20} P_{20}} \left(P_{20} - P_1 + \frac{F_{n0}}{A_1} \right) + B. \tag{9}$$

Thus,

(i) if $a_1 > 0$, then the system (7) is stable and velocity responses become damped oscillation.

(ii) if $a_1 < 0$, then the system (7) is unstable and velocity responses diverge and become self-excited oscillation.

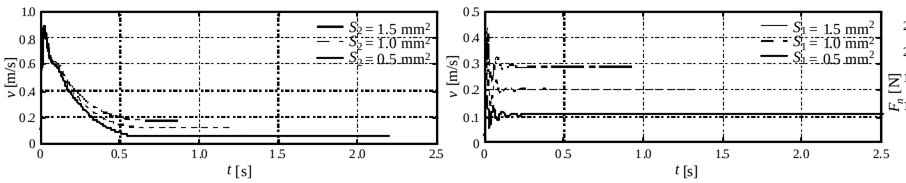
3.3 Meter-In Circuits

In the case of the meter-in circuit, we assume that the pressure in the rod-side cylinder chamber is equal to atmospheric pressure and that a supply-side restrictor reaches critical condition. Following the same process as with the meter-out circuit, a dynamic model is derived as

$$\frac{d^2(\Delta v)}{dt^2} + \frac{b_1}{m} \cdot \frac{d(\Delta v)}{dt} + \frac{b_2}{m} \cdot \Delta v = 0 \tag{10}$$

where $b_1 = bm + B,\ b_2 = k A_1^2 P_{10} \big/ V_{10} + b B,\ b = k A_1 v_0 \big/ V_{10}$. Therefore, the sign of the damping coefficient of the system (10) depends on

$$b_1 = \frac{k m R T_s C_{d1}}{V_{10} P_{10}} P_s + B. \tag{11}$$



(a) (b)

Figure 2: Simulation results

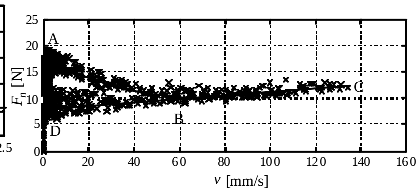


Figure 3: Frictional properties

3.4 Simulation Results

Figure 2 shows the simulation results. In Fig. 2a, the velocity fluctuates considerably and, on the contrary, Figure 2b presents slight oscillation and high attenuation. In addition, the velocity is dependent on the magnitude of the effective sectional area of restrictors in both cases.

It is worth noting that, although the friction force is defined by (4) in the above analysis, actual friction-velocity characteristics in a low-velocity range exhibit nonlinearity, as appears in Fig. 3. Therefore, with low velocity, the frictional coefficient B is negative and its magnitude varies remarkably. If B becomes negative, then a large, self-excited oscillation occurs in both driving circuits due to the negative damping coefficient from (9) and (11). As a result, we can recognize that such variation of the frictional coefficient causes transformation of oscillation.

4 EXPERIMENTS

4.1 Experimental Conditions

Taking account of the above analysis, we examined the manner of oscillation of velocity with the apparatus shown in Fig. 1 when the amount of valve opening changes. When pneumatic cylinders work at extremely low speed, depending on operating conditions the velocity fluctuates randomly, and that sometimes turns into stick-slip or chaotic oscillation. In particular, with respect to chaotic oscillation, we tried to specify the regions where it would occur in relation to the effective sectional area of valves, in order to investigate its factors. In the experiments, three chaos indices proposed by Ott, E. (1981) are used for judging the chaotic characteristics: power spectrum, auto-correlation function, and maximum Lyapunov exponent (Wolf, A.; Swift, J.B.; Swinny, H.L. (1985)).

The effective sectional area of the supply-side valve, S_1, was set from 2.45 mm^2 to 3.70 mm^2 at a pitch of 0.05 mm^2; that of the rod-side valve, S_2, was set from 0.05 mm^2 to 4.40 mm^2 at a pitch of 0.15 mm^2. Five measurements per one measured point were made, and the measured values were recorded at a sampling rate of 1 kHz.

4.2 Experimental Results

The result acquired under the condition of P_s = 0.25 MPa and m = 1.5 kg is illustrated in Fig. 4. The incidence of chaotic oscillation is plotted as gray scale, with S_2 as ordinate and S_1 as abscissa. White regions mean that the rate of occurrence is 0% and black regions mean that it is 100%. At the higher density, the incidence increases. An obliquely lined part corresponds to a stick-slip occurrence region. As Fig. 4 indicates, the rate is highest in the range of $S_1 > S_2$ and the neighborhood of the boundary between the stick-slip region and the non-stick-slip region.

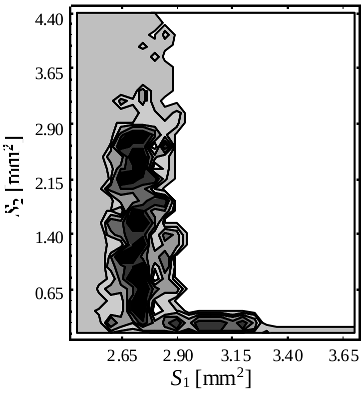


Figure 4: Measured results

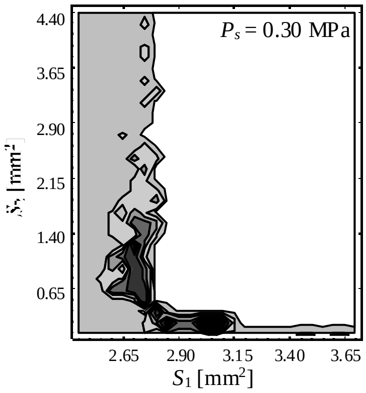


Figure 5: Measured results

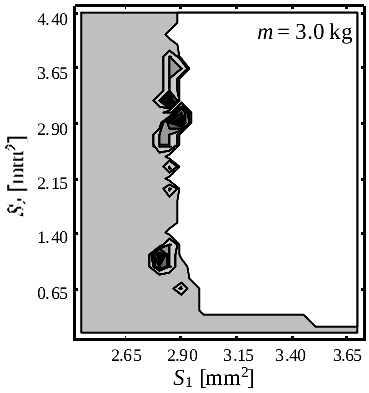


Figure 6: Measured results

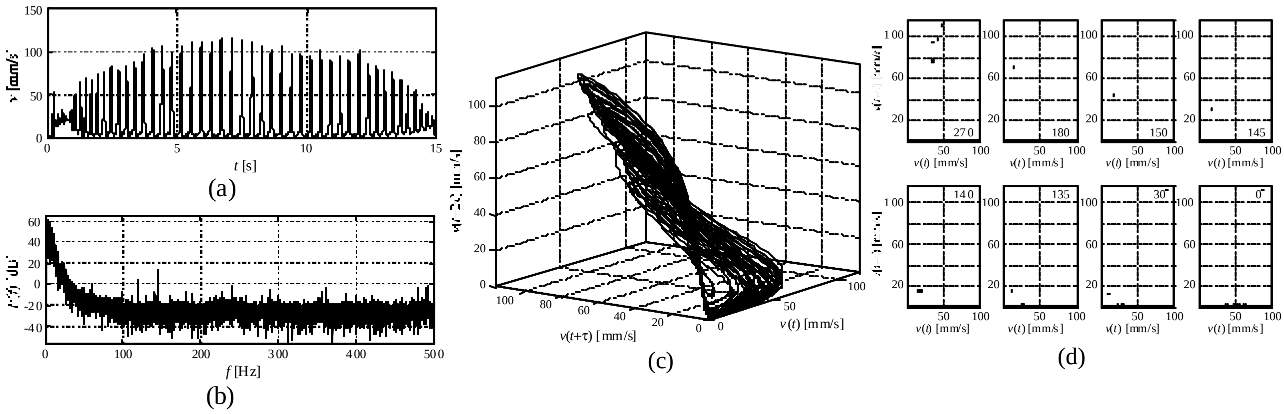


Figure 7: An example of chaotic oscillation

The result of testing the effect of supply pressure changes is shown in Fig. 5 (P_s = 0.30 MPa, m = 1.5 kg). In comparison with Fig. 4, the stick-slip region indicated by the oblique lines decreases by about 20%, and the region in which the chaotic oscillation happens at a rate of 100% also contracts by more than half. Contrary to this, increased supply pressure extends the stick-slip region and simultaneously reduces the chaotic oscillation region.

The result of testing the influence of load changes is shown in Fig. 6 (P_s = 0.25 MPa, m = 3.0 kg). A doubled load promotes stick-slip occurrence, and there are a few points where chaotic oscillation always happens, as seen in Fig. 6. A tripled load completely prevents chaotic behavior.

Figure 7a exemplifies a velocity response having chaotic behavior (S_1 = 2.70 mm^2, S_2 = 0.35 mm^2). Figure 7b shows a power spectrum to the time series of Fig. 7a, and it exhibits wide and even distribution without dominant peaks. A value of the auto-correlation function to Fig. 7a converged to zero. The maximum Lyapunov exponent converged to a positive value (3.41). In addition to these three indices, a trajectory on three-dimensional space and its Poincare section are obtained as shown in Fig. 7c and Fig. 7d, respectively. The trajectory is folded between 180 and 140 and is stretched between 140 and 0. These results also support chaotic characteristics of oscillation.

In Figs. 4 ~ 6, chaotic oscillation occurs in the limited region in which S_1 and S_2 are relatively small. Since it also occurs in the vicinity of the border between the stick-slip and non-stick-slip regions, it is found that the velocity oscillates chaotically when it is particularly low and its motion does not fall into stick-slip. Closely investigating frictional characteristics around there, it can be seen that the friction force makes a loop, such as $A \rightarrow B \rightarrow C \rightarrow B \rightarrow D \rightarrow A$ in Fig. 3, and that it greatly fluctuates without a standstill. Namely, as mentioned in Section 3.4, the frictional coefficient B practically varies in sign and magnitude, and the oscillation transforms from self-excited oscillation to damping oscillation, or vice-versa. Therefore, we experimentally confirmed that such transformation causes random oscillation or chaotic oscillation. Moreover, from Fig. 5 and (11), increased supply pressure improves damping capacity of the whole system. From Fig. 6, additional loads increase a damping coefficient in (9) and (11) and prevent chaotic behavior. Consequently, chaotic oscillation principally depends on nonlinear characteristics of the friction force.

5 CONCLUSION

The main results derived from this study are summarized as follows:

1) We have shown the chaotic oscillation occurrence regions and stick-slip regions in relation to the effective sectional area of the supply-side or rod-side valve of a pneumatic cylinder. The influence of changes in load or in supply pressure on these regions has also been shown.
2) From the analytical approach with the model and the experimental study, it has been verified that the behavior of the velocity turns random or chaotic in a low-velocity range due to the nonlinearity of the friction force.

6 LIST OF NOTATIONS

A	Cross sectional area	m^2	B	Viscous frictional coefficient	Ns/m
C_d	Discharge coefficient	$kg/(s \cdot Pa)$	F_c	Coulomb friction force	N
F_n	Friction force	N	F_s	Static friction force	N
m	Total mass	kg	P	Pressure	Pa
R	Gas constant	$J/(kg \cdot K)$	S	Effective sectional area	m^2
T	Temperature	K	v	Piston velocity	m/s
V	Volume	m^3	W	Mass flow rate	kg/s
x	Piston position	m	k	Specific heat ratio	
0	Equilibrium state		1	Supply side	
2	Rod side		a	Atmosphere	
s	Supply				

7 REFERENCES

Belforte G.; Raparelli, T.; Velardocchia, M. (1993). Study of the behavior of lip seals in pneumatic actuators. *J. Tribologists and Lubrication Engineers, Vol. 49, No. 10, pp. 775-780.*

Schroeder, L.E.; Singh, R. (1993). Experimental study of friction in a pneumatic actuator at constant velocity. *ASME J. Dynamic Systems, Measurement, and Control, Vol. 115, pp. 575-577.*

Kawakami, Y.; Noguchi, H.; Kawai, S. (1990). Some Considerations on the High-Speed Driving of Pneumatic Cylinders. *Journal of JHPS, Vol. 21, No. 3, pp. 124-130. (in Japanese)*

Ott, E. (1981). Strange attractors and chaotic motions of dynamical systems. *Reviews of Modern Physics, Vol. 53, No. 4, pp. 655-671.*

Wolf, A.; Swift, J.B.; Swinny, H.L. (1985). Determining Lyapunov exponents from a time series. *Physica, Vol. 16D, pp. 285-317.*